

Lecture 23 - Nov 27

Recursion

Examples: reverseOf, occurrencesOf
Recursion on Arrays: Creating Sub-Arrays
Recursion on Arrays: Call by Value

Announcements/Reminders

- Today's class: [notes template](#) posted
- **Lab5** released (deadline: Dec 2; **grace period** until Dec 9)
 - + Required background study: **abstract classes** & **interfaces**

Recursions on Strings

not a palindrome.

already diff $\rightarrow$ false.

Palindrome

1st & last chars same

there's a change the substring, minus 1st & last char, is a Palindrome.

"racecar"

"aracecar"

"racedcar"

Reversal

rev("abcd")
 $\downarrow$
"dcba"

reverse of.

Number of Occurrences

occ("abca", 'a')

'a'

'b'

"abca"

$1 + \text{occ}("bcb", 'a')$

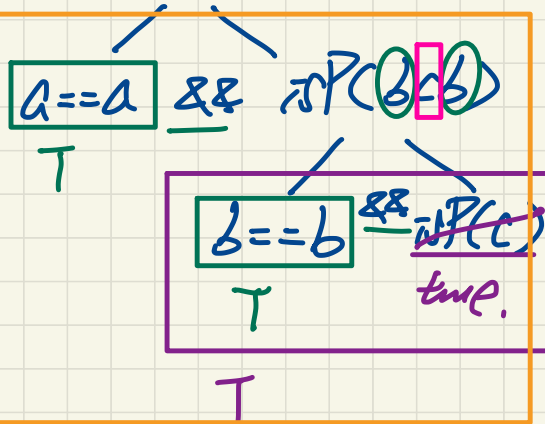
$0 + \text{occ}("bcb", 'b')$

Problem: Palindrome

```

boolean isPalindrome(String word) {
    if (word.length() == 0 || word.length() == 1) {
        /* base case */
        return true;
    }
    else {
        /* recursive case */
        char firstChar = word.charAt(0);
        char lastChar = word.charAt(word.length() - 1);
        String middle = word.substring(1, word.length() - 1);
        return
            firstChar == lastChar
            /* See the API of java.lang.String.substring. */
            && isPalindrome(middle);
    }
}
    
```

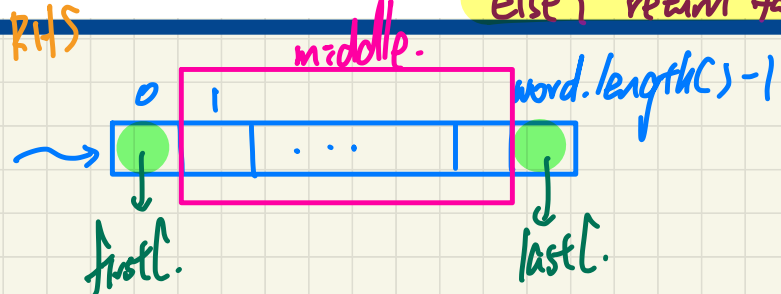
isP(abcb a)



if (fc == lc) {
return isP(middle);
} else { return false; }

recursive case.

1) LHS is true → eval RHS
2) LHS is false → skip RHS

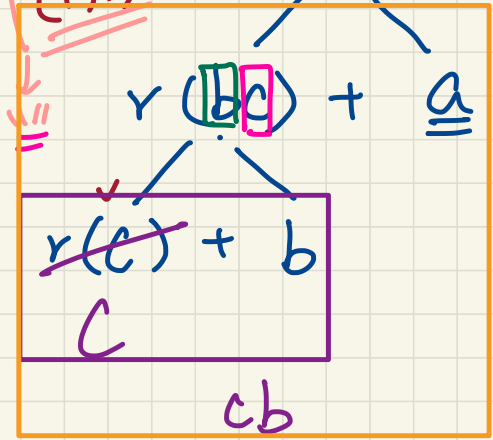


Problem: Reverse of a String

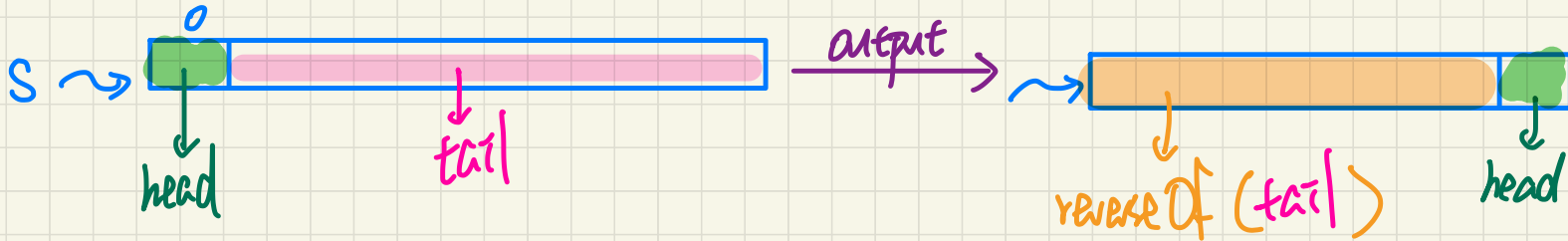
base cases

```
String reverseOf (String s) {  
    if (s.isEmpty()) { /* base case 1 */  
        return "";  
    }  
    else if (s.length() == 1) { /* base case 2 */  
        return s;  
    }  
    else { /* recursive case */  
        String tail = s.substring(1, s.length());  
        String reverseOfTail = reverseOf (tail);  
        char head = s.charAt(0);  
        return reverseOfTail + head;  
    }  
}
```

what if $s.length == 1$
"c".substring(1,1)



cba



a b c d

d c b a

alternative
approach
for implementing
reverse of.

Problem: Number of Occurrences

```
int occurrencesOf (String s, char c) {
    if (s.isEmpty()) {
        /* Base Case */
        return 0;
    }
    else {
        /* Recursive Case */
        char head = s.charAt(0);
        String tail = s.substring(1, s.length());
        if (head == c) {
            return 1 + occurrencesOf (tail, c);
        }
        else {
            return 0 + occurrencesOf (tail, c);
        }
    }
}
```

base.

$occ('ab', 'b')$

$a \neq b$
0

$occ('b', 'b')$

1

$b = b$
1

~~$occ('a', 'b')$~~
0



head

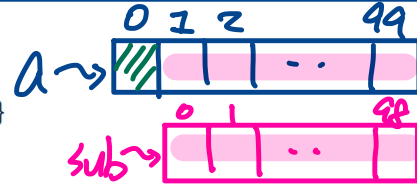
tail

1 if head == c + occ(tail, c)
0 if head != c

return head == c ? 1 + occ(tail, c) : occ(tail, c)

Recursion on an Array: Passing new Sub-Arrays

```
void m(int[] a) {  
    if(a.length == 0) { /* base case */ }  
    else if(a.length == 1) { /* base case */ }  
    else {  
        int[] sub = new int[a.length - 1];  
        for(int i = 1; i < a.length; i++) { sub[i - 1] = a[i]; }  
        m(sub) } }  
}
```

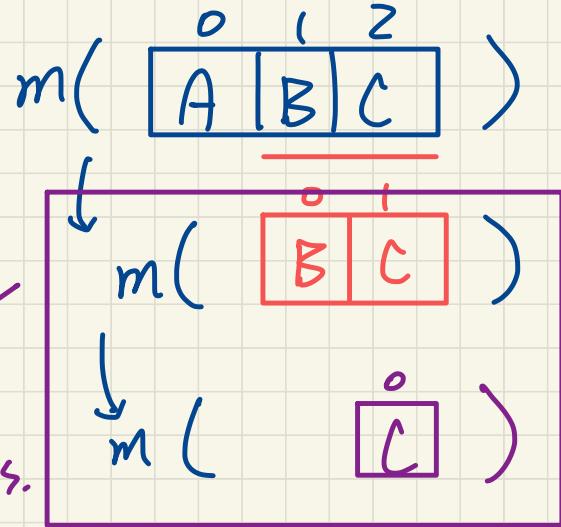


Say $a1 = \{\}$, consider $m(a1)$

↳ base case #1.

Say $a2 = \{A, B, C\}$, consider $m(a2)$

loop time & space wasted to create sub-arrays between R.C.s.



Recursion on an Array: Passing Same Array Reference

"sub-arrays"

range of indices

to inspect in array



```
void m(int[] a, int from, int to) {
    if (from > to) { /* base case */ }
    else if (from == to) { /* base case */ }
    else { m(a, from + 1, to) } }
```

3, 2
2, 2

sub-array of size 0

sub-array of size 1

recursive call

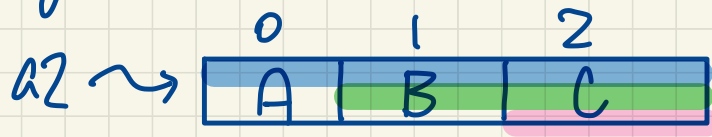
* call by value: passing the ref. of 'a' around.

Say $a1 = \{\}$, consider $m(a1, 0, a1.length - 1) \rightarrow m(a1, 0, -1)$

$a1.length == 0$
0 2
1st index last index

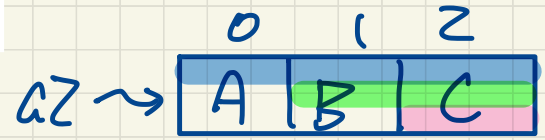
Say $a2 = \{A, B, C\}$, consider $m(a2, 0, a2.length - 1)$

$a2.length == 3$



$from > to$
empty range of indices to look at
 $\rightarrow$ empty sub-array

Say $a2 = \{A, B, C\}$, consider $m(a2, \underline{0}, \underline{a2.length - 1})$



$m(a2, \underline{0}, \underline{2})$

↳ input array of length 3

$m(a2, \underline{1}, \underline{2})$

↳ sub-array of length 2

$m(a2, \underline{2}, \underline{2})$

↳ base case (as if sub-array of length 1).

As recursive calls are made, narrow down the intervals.